

of the cumulative effect of rainfall is discovered. Assuming rainfall to be general over the watershed, each tributary in contributing its waters should serve to swell, by so much surplus as it carried, the total volume of the main stream, and this augmentation would, in turn, be shown by river gage readings made at or immediately below the point where it joined the main stream. Figuring on this basis and using maximum freshet data of from six to seventeen years, the average increase of freshet water carried by the James River would result, indicating what might be termed the discharge value of single or grouped tributaries. Thus, an average freshet height of 9.5 feet, representing the discharge of Dunlops Creek and Jackson River, is increased to 13.7 feet at Buchanan from the outflow of the Cowpasture River and Craigs and Catawba creeks; to 15.7 feet at Lynchburg from the waters of North River and smaller streams; to 18.9 feet at Scottsville by the Pedlar, Tye, and Rockfish rivers and smaller streams; and to 27.3 feet at Columbia by the State and Rivanna rivers. It is to be noted, however, that the local conditions at Columbia are such as to prevent a true increase or discharge value from being observed. The Rivanna River, the main tributary of the lower basin, here enters the James River at right angles, and the latter being narrow and shallow at this point becomes congested and a piled up condition of the water results that gives the local reading a value in excess of what it actually should be. Making allowance for this condition, it is probable that the ratio of increase at Columbia would be but slightly greater than that for Scottsville.

#### STUDIES ON THE CIRCULATION OF THE ATMOSPHERES OF THE SUN AND OF THE EARTH.

By Prof. FRANK H. BIGELOW.

#### IV.—VALUES OF CERTAIN METEOROLOGICAL QUANTITIES FOR THE SUN.

##### THE IMPORTANCE OF THESE VALUES TO TERRESTRIAL METEOROLOGY.

The most important data needed for use in studies in solar physics are the correct values of the pressure, the temperature, the density, the gas constant, and their many derived relations, at the surface of the sun, within its mass, and throughout the gaseous envelope. In the present uncertain state of our knowledge of these quantities, even an approximate derivation of these data is important, and this forms the justification for the studies contained in this paper. The problems of the circulation within the sun's photosphere, the transitions and the transformations in the atmospheric envelope with the attendant radiations and absorptions, the heat and light received at the outer surface of the earth's atmosphere, the resulting absorption and transmission of energy in the air, and the dependent circulation, are all languishing for the lack of a sound footing for our computations and deductions. The computations for the surface temperature of the sun give results ranging from 5000° to 10,000°; using Ritter's Law, Professor Schuster computes the temperature at the center of the sun as 12,000,000°, assuming that it is composed of hydrogen split up into monatomic elements. But it is evident that any such range of temperature would simply explode the sun, whereas it now circulates in a moderate manner. Unless some value for the temperature of the solar photosphere can be found, it will be impossible to determine what percentage of the total solar radiation is absorbed in the solar envelope, even though the radiant heat be computed successfully on the outer surface of the earth's atmosphere from radiation measurements at the ground. Should the following remarks prove to be merely suggestive it will be proper to make them as a contribution to the problems in solar physics.

I have been interested in the paper by Prof. F. E. Nipher, on the "Law of contraction of gaseous nebulae,"<sup>25</sup> because it seems to offer a way of escape from the impossible results

which follow from Ritter's equations, where the exponent in  $P v^n = B$  is 1.33+. Nipher makes the value of  $n = 1.10$ , and from this exponent the entire system of relations seems to be more probable. I will recapitulate Nipher's equations, after making the following changes in his notation to reduce them to the symbols used in my papers:

	Nipher.	Bigelow.
Gas constant	change $C$ to $R$	
Density	" $\delta$ "	" $\rho$ "
Distance from center	" $R$ "	" $r$ "
Mechanical equivalent of heat	" $J$ "	" $A' = \frac{1}{A}$ "
Heat equivalent of work	" $\frac{1}{J}$ "	" $A = \frac{1}{A'}$ "
Constant	" $A$ "	" $B$ "
Ratio	" $\rho$ "	" $b$ "
Constant	" $k$ "	" $k^2$ "

##### NIPHER'S EQUATIONS.

Adiabatic law for perfect gases:

$$(41) \quad P v = R T.$$

Heat relation:

$$(42) \quad dQ = c_v dT + P dv.$$

Assumed laws for non-perfect gases:

$$(43) \quad P v^n = P_0 v_0^n = B.$$

$$(44) \quad T v^{n-1} = \frac{B}{R}.$$

$$(45) \quad \frac{T^n}{P^{n-1}} = \frac{B}{R^n}.$$

Specific heat:

$$(46) \quad \left( \frac{dQ}{dT} \right)_n = c_v + \frac{AR}{1-n} \quad A = \frac{1}{4.19 \times 10^7}.$$

Gravitation:

$$(47) \quad \frac{dP}{dr} = -k^2 \frac{M}{r^2} \rho = -k^2 \frac{M}{r^2} \left( \frac{P}{B} \right)^{\frac{1}{n}} \quad k^2 = \frac{1}{1.5173 \times 10^7}.$$

Pressure:

$$(48) \quad P = \left[ \frac{4n-3n^2}{(2-n)^2} \cdot \frac{B^n}{2\pi k^2 r^2} \right]^{\frac{n}{2-n}} = \left[ \frac{0.95 B^{1.82}}{2\pi k^2 r^2} \right]^{1.22} \\ = \frac{0.95 R^2 T^2}{2\pi k^2 r^2} = \frac{0.636 M^2 k^2}{8\pi r^4}.$$

Density:

$$(49) \quad \rho = \left[ \frac{4n-3n^2}{(2-n)^2} \cdot \frac{B}{2\pi k^2 r^2} \right]^{\frac{1}{2-n}} = \left[ \frac{0.95 B}{2\pi k^2 r^2} \right]^{1.11} \\ = 0.95 \frac{R T}{2\pi k^2 r^2} = \frac{0.78 M}{4\pi r^3}.$$

Temperature:

$$(50) \quad T = \frac{B^{\frac{1}{2-n}}}{R} \left[ \frac{4n-3n^2}{(2-n)^2} \cdot \frac{1}{2\pi k^2 r^2} \right]^{\frac{n-1}{2-n}} = \frac{B^{1.11}}{R} \left( \frac{0.95}{2\pi k^2 r^2} \right)^{0.111} \\ = 0.818 \frac{M k^2}{2 R r}.$$

Mass:

$$(51) \quad M = 4\pi \left( \frac{2-n}{4-3n} \right) \left[ \frac{B(4n-3n^2)}{2\pi k^2 (2-n)^2} \right]^{\frac{1}{2-n}} r^{\frac{4-3n}{2-n}} \\ = 5.14\pi \left( \frac{0.95 B}{2\pi k^2} \right)^{1.11} r^{0.77} \\ = \frac{n}{2-n} \cdot \frac{2r R T}{k^2} = 1.22 \frac{2 R T r}{k^2}.$$

<sup>25</sup>Transactions Academy of Science, St. Louis, October 1, 1903.

Weight of one gram at the surface:

$$(52) \quad g = \frac{k^2 M}{r^2} = 4\pi k^2 \left( \frac{2-n}{4-3n} \right) \left[ \frac{B(4n-3n^2)}{2\pi k^2(2-n)^2} \right]^{\frac{1}{2-n}} \frac{1}{r^{\frac{n}{2-n}}} \\ = 5.14\pi k^2 \left( \frac{0.95B}{2\pi k^2} \right)^{\frac{1.11}{1.22}} r^{\frac{1}{1.22}} \\ = \frac{n}{2-n} \cdot \frac{2R}{r} T = 1.22 \frac{2R}{r} T.$$

Auxiliaries:

$$(53) \quad B = P v^n = \frac{k^2}{2n} \left( \frac{4\pi}{4-3n} \right)^{\frac{1}{2}} (2-n)^{\frac{4}{3}} M^{\frac{2}{3}} \\ = \left[ \frac{2\pi k^2}{(4-3n)n} \right]^{\frac{1}{3}} \left[ (2-n) R T r \right]^{\frac{2}{3}}.$$

$$(54) \quad B' = \left[ \frac{(4n-3n^2)B}{(2-n)^2 2\pi k^2} \right]^{\frac{1}{2-n}}.$$

Contraction ratio  $b = \frac{r_0 \text{ (initial)}}{r \text{ (final)}}$ :

$$\begin{cases} (55) & P = P_0 \left( \frac{r_0}{r} \right)^4 = P_0 b^4 \\ (56) & \rho = \rho_0 \left( \frac{r_0}{r} \right)^3 = \rho_0 b^3 \\ (57) & T = T_0 \left( \frac{r}{r_0} \right) = T_0 b^{-1}. \end{cases}$$

Mass:

$$(58) \quad M = \frac{4}{3} \pi r^3 \cdot \rho_a = 4\pi \frac{2-n}{4-3n} \rho \cdot r^3.$$

Average density:

$$(59) \quad \rho_a = 3 \frac{2-n}{4-3n} \frac{B'}{r^{\frac{2-n}{2}}} = 3 \frac{2-n}{4-3n} \rho = \dots\dots\dots 3.86 \rho$$

Distance from center to stratum where the density  $\rho =$  average density  $\rho_a$ :

$$(60) \quad r_a = \left[ \frac{4-3n}{3(2-n)} \right]^{\frac{2-n}{2}} r = \dots\dots\dots 0.545 r$$

Average pressure:

$$(61) \quad P_a = \frac{4\pi \int_0^r r^2 P dr}{4\pi \int_0^r r^2 dr} = 3 \frac{2-n}{6-5n} P = \dots\dots\dots 5.40 P$$

Distance from center to stratum where the pressure  $P =$  average pressure  $P_a$ :

$$(62) \quad r_a = \left[ \frac{1}{3} \cdot \frac{6-5n}{2-n} \right]^{\frac{2-n}{2n}} r = \dots\dots\dots 0.502 r$$

Average temperature:

$$(63) \quad T_a = 3 \cdot \frac{2-n}{8-5n} T = \dots\dots\dots 1.08 T$$

Distance from center to stratum where the temperature  $T =$  average temperature  $T_a$ :

$$(64) \quad r_a = \left[ \frac{1}{3} \cdot \frac{8-5n}{2-n} \right]^{\frac{2-n}{2(n-1)}} r = \dots\dots\dots 0.707 r$$

Specific heat:

$$(65) \quad \frac{dQ}{dT} = \frac{c_p \frac{dP}{P} + c_v \frac{dv}{v}}{\frac{dP}{P} + \frac{dv}{v}} = \frac{c_p - n c_v}{1-n} = \frac{c_p}{\kappa} \left( \frac{\kappa - n}{1-n} \right).$$

Auxiliaries.

$$(66) \quad \left( \frac{dQ}{dT} \right)_n = c_p - A T \left( \frac{dv}{dT} \right)_v \left( \frac{dP}{dT} \right)_n \quad \frac{dP}{P} = -n \frac{dv}{v} \\ = c_p - A T \frac{R M}{P} \cdot \frac{R M}{v} \quad v^n dP = -n P v^{n-1} dv. \\ = c_p - A R M^2 \quad \frac{dP}{P} = -\frac{2n}{2-n} \frac{dr}{r} \\ = c_p + 4A R \quad \frac{dv}{v} = +\frac{2}{2-n} \frac{dr}{r} \\ = c_v + \frac{A R}{1-n} = -7.365 \text{ (reversing sign).}$$

$$(67) \quad n = \frac{2c_p + 4A R}{2c_p + 3A R} = \frac{6\kappa - 4}{5\kappa - 3} = 1.10.$$

$$(68) \quad c_p = A R \frac{3n-4}{2-2n} = A R \frac{\kappa}{\kappa-1} = A R \frac{4-3\kappa}{2\kappa-1}.$$

$$(69) \quad R T = P v = \frac{4}{3} \pi r^3 P = \frac{2-n}{n} \cdot \frac{M k^2}{2r} = 0.818 \frac{M k^2}{2r}.$$

Heat:

$$(70) \quad Q = \left( \frac{dQ}{dT} \right)_n (T - T_0) = -(c_p + 4A R) (T - T_0) \\ = -(c_p + 4A R) T_0 (b - 1).$$

Work:

$$(71) \quad W = \int P dv = P_0 v_0^n \int_{v_0}^v \frac{dv}{v^n} = \frac{P_0 v_0}{1-n} (b - 1), \\ = \frac{A R T_0}{1-n} (b - 1) = -(c_v + c_p + 4A R) T_0 (b - 1), \\ = 4\pi \int_r^\infty r^2 P dr = \frac{4-3n}{n} \frac{M^2 k^2}{2r} = 0.636 \frac{M^2 k^2}{2r}.$$

Ratios:

$$(72) \quad c = \frac{Q}{W} = \frac{c_p + 4A R}{c_v + c_p + 4A R} = \frac{c_p + 4A R}{2c_p + 3A R} = \frac{5\kappa - 4}{5\kappa - 3} = 0.75.$$

$$(73) \quad \frac{Q}{W-Q} = \frac{c_p + 4A R}{c_p - A R} = \frac{c_p + 4A R}{c_v} = \dots\dots\dots 3.00.$$

$$(74) \quad \frac{W}{W-Q} = 5\kappa - 3 = \dots\dots\dots 4.00.$$

Differences:

$$(75) \quad c_p - c_v = \frac{8-5n}{3(2-n)^2} \cdot \frac{4-3n}{5\kappa-3} \cdot A R = \dots\dots\dots 0.180 A R \\ = \frac{1}{5\kappa-3} \cdot \frac{4-3n}{n} \cdot \frac{M k^2 A}{2r T_a}.$$

For a rise of  $1^\circ \text{C.}$ , energy equivalent to  $2c_p + 3A R$  heat units must be applied to the unit mass, of which  $c_p + 4A R$  heat units are radiated per unit time, and  $c_p - A R = c_v$  heat units are used in raising the temperature.

THE ASTRONOMICAL CONSTANTS FOR THE EARTH AND THE SUN.

It is difficult to select from the available astronomical data a system of constants that is rigorously self-consistent, and in this preliminary discussion it is not necessary to make complete adjustments between the several quantities. The fundamental units employed are conveniently the C. G. S. system, and not the C. S. system, because in the thermodynamic formulæ the unit of mass is the gram. In the C. G. S. system the gravitation constant is found from the formula,

$$(76) \quad g_0 = k^2 \frac{M_1 m}{R_1^2}, \text{ so that, } k^2 = \frac{R_1^2 g_0}{M_1 m}.$$

The constant for transformations from the C. S. system to the C. G. S. system is  $\frac{1}{k^2}$ ; i. e., (mass C. S.)  $\frac{1}{k^2} =$  (mass C. G. S.).

TABLE 7.—*Astronomical constants.*

	Numbers.	Logarithms.
$R_1$ = mean radius of earth, <i>Bessel's</i> spheroid	6370 19100 cm.	8. 8041525
$R_1^2$ =		17. 6083050
$R_1^3$ =		26. 4124575
$\rho_{a1}$ = average density of earth, <i>Harkness</i>	5. 576	0. 746323
$\frac{4}{3} \pi =$	4. 1888	0. 622089
$M_1 = \frac{4}{3} \pi R_1^3 \rho_{a1}$ = mass of the earth in grams	$6. 0377 \times 180^{27}$	27. 78070
$m$ = 1 gram	1. 00	0. 000000
$g_0$ = acceleration per second at surface of earth	980. 60 cm.	2. 991492
$k^2 = \frac{R_1^2 g_0}{M_1 m}$ = constant	$\frac{1}{1. 5173 \times 10^7}$	2. 818927—10
$\frac{1}{k^2}$ = transformation constant	$1. 5173 \times 10^7$	7. 181073
$\frac{M}{M_1}$ = ratio of mass of sun to mass of earth, <i>Newcomb</i>	333432.	5. 523008
$M$ = mass of the sun	$2. 0132 \times 10^{33}$	33. 303878
$r$ = radius of sun for <i>Auver's</i> di- ameter (31' 59.26")	694800 80000 cm.	10. 8418603
$r^2$ =		21. 6837206
$r^3$ =		32. 5255809
$p$ = parallax of the sun, <i>Newcomb</i> .	8. 7965"	0. 9443099
$D$ = distance from sun to earth	1493 40870 00000 cm.	13. 1741786
$r/R_1$ = ratio of radii	109. 071	2. 0377078
$S/S_1$ = ratio of surfaces (109. 071) <sup>2</sup>	11896. 4	4. 0754156
$V/V_1$ = ratio of volumes (109. 071) <sup>3</sup>	1297548.	6. 1131234
$G = \frac{\text{gravity at surface of sun}}{\text{gravity at surface of earth}} = \frac{M}{M_1} \cdot \frac{R_1^2}{r^2}$	28. 028	1. 4475924
$\rho_s$ = mean density of the sun = $\frac{M}{M_1} \cdot \frac{R_1^3}{r^3} \cdot \rho_{a1}$	1. 43287	0. 156208
$v$ = velocity of the earth in its orbit	$\left\{ \begin{array}{l} 18. 5212 \text{ miles/sec.} \\ 29. 80670 \text{ cm./sec.} \end{array} \right.$	$\left\{ \begin{array}{l} 1. 267670 \\ 6. 474314 \end{array} \right.$
$f = \frac{v^2}{D}$ = acceleration at the dis- tance of earth.	$\left\{ \begin{array}{l} 0. 59491 \text{ cm./sec.} \\ 0. 23422 \text{ inch/sec.} \end{array} \right.$	$\left\{ \begin{array}{l} 9. 774448-10 \\ 9. 369615-10 \end{array} \right.$
$f = \frac{M}{M_1} \left( \frac{R_1}{D} \right)^2 g_0$ (check)		9. 77448—10
$s = \frac{1}{2} f$ = rate at which earth falls toward sun.	$\left\{ \begin{array}{l} 0. 29746 \text{ cm./sec.} \\ 0. 11711 \text{ inch/sec.} \end{array} \right.$	$\left\{ \begin{array}{l} 9. 473418-10 \\ 9. 068585-10 \end{array} \right.$

APPLICATION OF THE THERMODYNAMIC FORMULÆ TO THE GASEOUS ENVELOPE OF THE SUN.

The evidence from the action of the lines in the solar spectrum, as regards shifting, broadening, and reversals, shows that in the envelope resting upon the photosphere, comprising in its contents the reversing layer, the chromosphere, and the inner corona, the gases may be treated as approximately perfect gases and tending to conform to the Boyle-(Mariotte)-

Gay Lussac law,  $P v = \frac{K}{m} T$ , where  $P$  is the pressure in units of force,  $v$  the volume,  $K$  the absolute gas constant,  $m$  the molecular weight, and  $T$  the absolute temperature. I propose, also, to apply the same law to the solar mass within the photosphere, with a suitable modification, and to compare the results with the data obtained from the use of Professor Nipher's equations. We can first multiply the equation by any numerical value,  $x$ , and distribute the variation between  $P$  and  $T$  alone, holding the density identical in the two conditions.

Hence,

(77)  $(x P) v = \frac{K}{m} (x T).$

This asserts that if  $\frac{K}{m}$  remains constant,  $v = \frac{1}{\rho}$  also remains constant. If a gas, as hydrogen,  $\rho = 0.000089996$ , is subjected to the same relative increase in  $P$  and  $T$ , it remains at the same density as that for which its gas constant  $\frac{K}{m}$  was computed.

We can, therefore, transform hydrogen, or other perfect gases, from terrestrial to solar conditions by simply multiplying by the proper factor. In this case it will be  $x = 28.028$ , the ratio of  $g$  at the surface of the sun to  $g_0$  at the surface of the earth.

In Eclipse Meteorology and Allied Problems, chapter 4, Table 14.—“Fundamental constants,” a series of values was computed depending upon assumed values of  $R$ , the sun's radius, and  $G$ , the ratio between gravity at the surface of of the sun and gravity at the surface of the earth. Since these values have been changed a little in the preceding computations, it will be necessary to reconstruct the numerical values of that table, although the effect upon the dependent quantities is not important. In order that the transition from terrestrial to solar conditions may be made as plain as possible to the reader, we will compute the fundamental constants on the supposition that the earth is surrounded by a hydrogen atmosphere instead of the common air, making allowance for the change in density.

TABLE 8.—*Constants for one atmosphere of hydrogen on the earth.*  
 $p_0 v_0 = R T_0 = l.$

Formulæ.	M. K. S. system.		C. G. S. system.	
	Numbers.	Loga- rithms.	Numbers.	Loga- rithms.
$g_0$ = gravity	9. 806 m.	0. 99149	980. 6 cm.	2. 99149
$\rho_m$ = density of mercury	13595. 8	4. 13340	13. 5958	1. 13340
$B_u$ = merc. col. for 1 atmos	0. 760	9. 88081	76. 0	1. 88081
$\rho_u$ = density of hydrogen	0. 089996	8. 95422	0. 000089996	5. 95422
$p_0 = \rho_m B_u = \rho_u l$ (weight)	10333.	4. 01421	1033. 3	3. 01421
$l = \frac{\rho_m B_u}{\rho_u}$ (hom. atmos)	114815.	5. 05999	11481500.	7. 05999
$T_0$ = temperature	273.	2. 43616	273.	2. 43616
$R_0 = \frac{l}{T_0}$ = gas constant	420. 56	2. 62383	42056.	4. 62383
$v_0 = \frac{1}{\rho_u}$ = specific volume	11. 112	1. 04578	11112.	4. 04578
$p_0 v_0 = l$	114815.	5. 05999	11481500.	7. 05999
$R_0 T_0 = l$	114815.	5. 05999	11481500.	7. 05999

TABLE 9.—*Transition to constants for a solar hydrogen atmosphere.*  
 $(G p_0) v_0 = R (T_0 G).$

Formulæ.	M. K. S. system.		C. G. S. system.	
	Numbers.	Loga- rithms.	Numbers.	Loga- rithms.
$G = g/g_0$	28. 028	1. 44759	28. 028	1. 44759
$G p_0 = p$ (weight)	289600.	5. 46180	28960.	4. 46180
$v_0 = v$ (same density)	11. 112	1. 04578	111112.	4. 04578
$p v = l$	3218000.	6. 50758	321800000.	8. 50758
$G T_0 = T$	7651. 6	3. 88375	7651. 6	3. 88375
$R_0 = R$	420. 56	2. 62383	42056.	4. 62383
$R T = l$	3218000.	6. 50758	321800000.	8. 50758

TABLE 10.—*Fundamental constants for a hydrogen atmosphere on the sun.*

Data.	Formulæ.	Meter-kilogram-second.		Centimeter-gram-second.	
		Number.	Logarithm.	Number.	Logarithm.
Radius of the sun	$r$	694800800 m	8.8418603	694800 80000 cm	10.8418603
Gravity acceleration at the surface	$g = G g_0 = 28.028 \times 9.806$	274.843	2.4390843	27484.3	4.4390843
Modulus of common logarithms	$M$	0.4342945	9.6377843—10	0.4342945	9.6377843—10
Density	Mercury $\rho_m$	13595.8	4.1334048	13.5958	1.1334048
	Water $\rho_l$	1000	3.0000000	1.0000	1.0000000
	Air $\rho_0$	1.29305	0.1116153	0.00129305	7.1116153—10
	Hydrogen $\rho_h$	0.089996	8.9542232—10	0.00089996	5.9542232—10
Height of standard barometer	$B_u = \frac{P}{\rho_m} = 0.760 \times 28.028$	21.3013	1.3284060	2130.13	3.3284060
Height of homogeneous atmosphere	$l = \frac{\rho_m B_u}{\rho_h} = R T = p v_h = \frac{P}{\rho_h}$	3218012	6.5075876	321801200	8.5075876
Barometric constant	$K = \frac{l}{M} = \frac{\rho_m B_u}{\rho_h M} = \frac{R T}{M}$	7409746	6.8698033	740974600	8.8698033
Pressure in units of weight	$p = \rho_h l = \rho_m B_u = \frac{P}{g}$	289608.1	5.4618108	28960.81	4.4618108
Pressure in units of force	$P = \left\{ \begin{array}{l} g \rho_h l = g \rho_m B_u = g p = P_0 G^2 = \\ g \rho_h p v_h = g \rho_h R T = \frac{\kappa}{\kappa-1} \frac{l}{T} = \frac{C_p}{A} \end{array} \right\}$	79596670.9	7.9008951	795966709	8.9008951
Press. of one terrestrial atmosphere	$P_0$	101323.5	5.0057103	1013235 dynes	6.0057103
Volume (specific) of hydrogen	$v_h = \frac{1}{\rho_h}$	11.1116	1.0457768	11111.6	4.0457768
Gas constant for pressure $p$	$R = \frac{p v_h}{T} = \frac{\rho_m B_u}{\rho_h T}$	420.565	2.6238330	42056.5	4.6238330
Gas constant for pressure $P$	$Rg = \frac{p v_h g}{T} = \frac{\rho_m B_u g}{\rho_h T}$	$1.15589 \times 10^5$	5.0629173	$1.15589 \times 10^9$	9.0629173
Temperature at the photosphere	$T = 28.028 \times 273$	7651.6° C.	3.8837546	7651.6° C.	3.8837546
Temperature gradient	$-\frac{dT}{dh} = \frac{A}{c_p} = \frac{1}{P}$	$1.2563 \times 10^{-8}$	2.0991049—10	$1.2563 \times 10^{-9}$	1.0991049—10
Specific heat at constant pressure	$c_p = g \rho_h A R T$	186503	5.2706707	18.9968	1.2791478
Heat equivalent of work	$A = \frac{1}{426.8} \text{ and } \frac{1}{4.1855} \times 10^7$	0.00234302	7.3697756—10	$2.38663 \times 10^{-8}$	2.3782527—10
Coefficient from specific heats	$\epsilon_p = \frac{\kappa}{\kappa-1} = g \rho_h T$	189261.5	5.2770621	18926.15	4.2770621
Ratio of the specific heats	$\kappa = \frac{c_p}{c_v}$	1.000005	0.0000021	1.000052	0.0000228

The constants are worked out for the meter-kilogram-second (M. K. S.) system and for the centimeter-gram-second (C. G. S.) system, respectively, the formulæ, which are well known, being found in Table 64 of the Report of the Chief of the Weather Bureau, 1898–99, Vol. II.

If hydrogen, as a perfect gas, conforms to the Boyle-Gay Lussac law at so high a temperature as 7651.6, then there must be some stratum in the sun's atmosphere where the density is the same as it is under the standard conditions on the earth. If the gas ceases to be perfect to some extent, this statement must be proportionately modified, but in any case even approximate conditions will be very valuable as giving a general view of the prevailing state of solar physics, in which a footing of some sort is a desideratum for meteorology in general. We next determine the temperature gradient by the computation in Table 10, in which the same constants are employed as above, except that their values have been determined with greater precision.

To obtain the temperature gradient per meter, or the adiabatic rate of fall of temperature per meter, the value of  $-\frac{dT}{dh}$

in the M. K. S. system must be multiplied by 1000, and in the C. G. S. system it must be multiplied by 10000 so that they both give

$$(78) \quad -\frac{dT}{dh} = 0.000012563^\circ \text{ C. per meter, or,}$$

$$(79) \quad -\frac{dT}{dh} = 0.012563^\circ \text{ C. per 1000 meters.}$$

This can be checked from the terrestrial adiabatic rate, which is 9.86938 per 1000 meters, by multiplying by  $\frac{1}{G^2}$ .

$$(80) \quad \text{Thus, } \left(-\frac{dT}{dh}\right)_{\text{sun}} = \left(-\frac{dT}{dh}\right)_{\text{earth}} \times \frac{1}{G^2}.$$

$$(81) \quad 0.012563 = 9.86938 \times \frac{1}{(28.028)^2}.$$

The rate of the fall in temperature in the atmosphere of the sun is very slow according to this computation, so that variation in the density of the gases is not due so much to changes in temperature as to changes in pressure, which are very rapid, as is shown in Table 11 and fig. 21. The approximate formula

is all that is necessary in this discussion because of the steady state of the temperature just indicated.

Let  $P_0$  = the pressure of 28.028 atmospheres, where  $h_0$ , the height, is assumed to be zero.

$P$  = the pressure in atmospheres at the height  $h$ .

$K$  = 7409.746 kilometers, the barometric constant.

Then we shall have the reduction formula:

(82)  $\log \frac{P_0}{P} = \frac{h-h_0}{K}$ , and  $\log P = \log P_0 - \frac{h}{K}$ .

The value of  $h$  in seconds of arc is found from

(83)  $1''$  (second of arc) =  $\frac{\text{radius of sun in kilometers}}{\text{radius of sun in seconds of arc}}$   
 $= \frac{694800.800}{16' \times 60 = 960''} = 723.751 \text{ km.}$

DISTRIBUTION OF THE PRESSURE, TEMPERATURE, AND DENSITY IN A SOLAR HYDROGEN ATMOSPHERE.

Since in a perfect gas  $Pv = \frac{P}{\rho} = RT$ , we shall have for the

density,  $\rho = \frac{P}{RT}$ . In order to compute  $R$ , the gas constant,

we take  $R = \frac{P}{\rho T}$ , where,

$P$  = 28.028 atmospheres,  
 $\rho$  = 0.089996,  
 $T$  = 7651.6°, whence we obtain  
 $R = 0.040702$  [logarithm = 8.6096146].

The values resulting from the computation are given in Table 11 and fig. 21, "Distribution of the pressure, temperature, and density in a solar hydrogen atmosphere." The indications regarding the prevailing pressure, derived from the behavior of certain lines in the solar spectrum, are that the reversing layer is under a pressure of about 5 atmospheres, or possibly as little as 3 atmospheres (Astrophysics, February, 1896, p. 139; May, 1898, p. 327; April, 1900, p. 240). According to Table 11 the pressure at the height 8'' above the stratum

TABLE 11.—Distribution of the pressure, temperature, and density in the solar hydrogen atmosphere.

$h''$ in arc.	$h$ in km.	$\frac{h}{K}$	$P$	$T$	$\rho$	Height of layer ( $h-7$ )'' above photo- sphere.
45	32568.75	4.39539	0.001	7242.4	0.000004	38
						Top of inner corona.
40	28950.00	3.90702	0.003	7287.9	0.000012	33
35	25331.25	3.41864	0.011	7333.4	0.000036	28
30	21712.50	2.93026	0.033	7378.8	0.000110	23
25	18093.75	2.44189	0.101	7424.3	0.000335	18
20	14475.00	1.95351	0.312	7469.7	0.001026	13
18	13027.50	1.75816	0.489	7487.9	0.001605	11
16	11580.00	1.56281	0.767	7506.1	0.002510	9
14	10132.50	1.36746	1.203	7524.3	0.003927	7
						Top of chromo- sphere.
12	8685.00	1.17210	1.886	7542.5	0.006143	5
10	7237.50	0.97676	2.957	7560.7	0.009609	3
9	6513.75	0.87908	3.703	7569.8	0.012018	2
8	5790.00	0.78140	4.636	7578.9	0.015031	1
						Reversing layer.
7	5066.25	0.68380	5.805	7587.9	0.018796	0
						Top of photo- sphere.
6	4342.50	0.58605	7.270	7597.0	0.023512	-1
5	3618.75	0.48838	9.104	7606.1	0.029406	-2
4	2895.00	0.39070	11.400	7615.2	0.036779	-3
3	2171.25	0.29303	14.275	7624.3	0.045999	-4
2	1447.50	0.19535	17.875	7633.4	0.057532	-5
1	723.75	0.09768	22.383	7642.5	0.071955	-6
0	0	0	28.028	7651.6	0.089998	-7
						Within the photosphere.

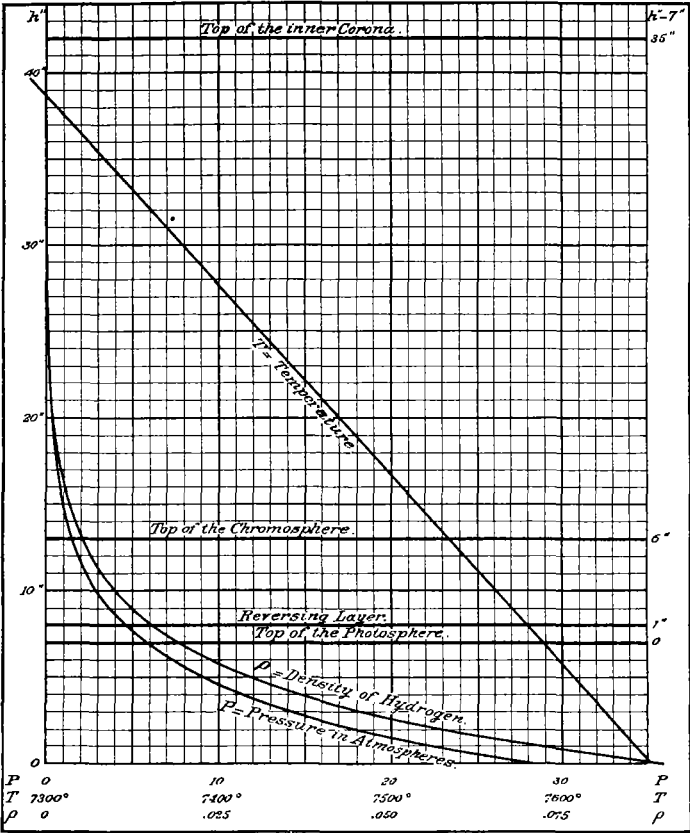


FIG. 21.—Distribution of the pressure, temperature, and density in a solar hydrogen atmosphere.

having a pressure of 28.028 atmospheres is 4.636, and this may be adopted as the height of the reversing layer. If the top of the photosphere is 1'' below the reversing layer, the top of the chromosphere 5'' above it, and the top of the inner corona 35'' above the top of the photosphere, then the layer at pressure 28.028 atmospheres is 7'' below the top of the photosphere, and is probably in the midst of the photospheric shell. The temperature gradient is a straight line,<sup>26</sup> but the pressure and density are distributed on curves of the logarithmic type. From 28 to 5 atmospheres the pressure and density change very rapidly, but from 2 to 0 atmospheres they change very slowly. There is a quick transition in the rate of change

<sup>26</sup> Since the temperature gradient,  $-\frac{dT}{dh} = \frac{1}{P_0}$ , was computed for the stratum within the photosphere, where  $P_0 = 28.028$ , it follows that in the higher strata, in which  $P$  has smaller values according to Table 11, the gradient will be greater in the proportion,  $\left(-\frac{dT}{dh}\right) \frac{P_0}{P}$ , assigning successive values to  $n$  in the several strata. Hence, the temperature fall is  $-\Delta T_n = -\int \left(\frac{dT}{dh}\right)_n dh$ , where  $\left(\frac{dT}{dh}\right)_n$  increases toward the top of the solar atmosphere. Computations for the successive layers show that the temperature fall is slow up to the level of  $P=1$  atmosphere, beyond which it increases very rapidly as the density diminishes, so that the temperature of space is reached at the top of the inner corona.

We obtained the following temperatures:

Initial stratum in photosphere	$P=28.028$ , $T=7652^\circ$
Top of the photosphere	$P=5.805$ , $T=7500^\circ$
Top of the reversing layer	$P=4.636$ , $T=7450^\circ$
Top of the chromosphere	$P=1.500$ , $T=6950^\circ$

This law gives too low temperatures at the top of the inner corona to be acceptable at present. Referring to the earth's atmosphere, the law of cooling is not the adiabatic rate, but the gradient is nearly the same as that found for the lower strata in all levels up to 16,000 meters: that is to say, cooling takes place at a uniform rate. The law of cooling in the solar atmosphere is a function which is not now known, and it may fall between the two extreme types indicated above. The entire subject demands a careful research.

between 5 and 2 atmospheres, and in the midst of this the reversing layer and chromosphere are located. It is, therefore, probable that the action in the reversing layer which sends forth visible light waves is due to rapid transmissions in pressure and density, rather than to any changes of temperature. This favors the theory proposed for the explanation of the reversing layer by Becquerel, Wood, and Julius, namely, that it is due to contrasts of density, and in accordance with which the phenomenon has been reproduced in the laboratory. Compare pages 65 and 162, *Eclipse Meteorology and Allied Problems*, Weather Bureau Bulletin I.

The shifting and the broadening of the lines in the spectrum are due to a variation of pressure and density rather than to a change of temperature. It is also seen that the density of the hydrogen approaches zero at the height of the top of the inner corona. The coincidence in the observed boundaries

TABLE 12.—*Computation of the pressures, temperatures, and densities at the surface and within the sun by Nipher's formulae.*

Fundamental constants.			
		Numbers.	Logarithms.
$M$	= total mass of the sun	$2.0132 \times 10^{33}$	33.303878
$\frac{1}{k^2}$	= gravitation constant	$1.5173 \times 10^7$	7.181073
$r$	= radius of the sun in centimeters	694800 80000.	10.841860
$T$	= absolute temperature at surface	7651.6°	3.883755
Density at surface and within the sun.			
$\rho$	$\left\{ \begin{array}{l} = \frac{0.78}{4\pi} \cdot \frac{M}{r^3} = \text{surface density} \\ = \frac{\rho_a}{3.86} = \frac{1.43287}{3.86} = \text{surface density} \end{array} \right.$	0.37255 0.37121	9.571182 9.569621
	$\rho_a$	= average density from astronomical data	1.43287
$r_a$	= 0.545 $r$ = distance of stratum $\rho_a$ from center	$3.7867 \times 10^{10}$	10.578257
$v$	= $\frac{1}{\rho}$ = specific volume at the surface	2.6842	0.428818
Pressure at the surface and within the sun.			
$P$	= $\frac{0.636}{8\pi} \cdot \frac{M^2 k^2}{r^4}$ = surface pressure	$2.9004 \times 10^{14}$	14.462460
$P_a$	= 5.40 $P$ = average pressure	$1.5662 \times 10^{15}$	15.194854
$r_a$	= 0.502 $r$ = distance of stratum $P_a$ from center	$3.4879 \times 10^{10}$	10.542564
$RT$ at the surface of the sun.			
$RT$	$\left\{ \begin{array}{l} = 0.818 \frac{Mk^2}{2r} \\ = P_v \text{ (The coefficient should be more fully developed.)} \end{array} \right.$	$7.8103 \times 10^{14}$ $7.7854 \times 10^{14}$	14.892668 14.891278
	$R$	= $\frac{P_v}{T}$ = the gas constant	$1.0175 \times 10^{11}$
Temperature at the surface and within the sun.			
$T$	= $273 \times 28.028$	7651.6°	3.883755
$T_a$	= 1.08 $T$	8263.8°	3.917179
$r_a$	= 0.707 $r$	$4.9122 \times 10^{10}$	10.691279
$-\Delta T$	= $8263.8^\circ - 7651.6^\circ = 612.2^\circ$	612.2°	2.786893
$+\Delta r$	= $1.000 r - 0.707 r = 0.293 r$ in km.	203577.	5.308728
$-\frac{\Delta T}{\Delta r}$	= temperature gradient within the sun per 1000 meters	0.0030072	7.478165—10
Mass of the sun.			
$M$	$\left\{ \begin{array}{l} = 1.22 \frac{2RTr}{k^2} = \text{mass} \\ = \text{Adopted value from Newcomb} \end{array} \right.$	$2.0091 \times 10^{33}$ $2.0132 \times 10^{33}$	33.302991 33.303878
	Weight of 1 gram at the surface of the sun.		
$g$	$\left\{ \begin{array}{l} = 1.22 \frac{2RT}{r^2} = M \frac{k^2}{r^2} \\ = 980.6 \times 28.028 \end{array} \right.$	27428. 27484.	4.438178 4.439084

of these layers in the sun's atmosphere with the results of this computation on the physical state is evidently so perfect as to argue strongly for the correctness of the physical constants employed. The outcome goes to show that the photosphere is the region where great changes in pressure are taking place, so that violent circulations, explosions, and chemical and electrical combinations must prevail, and observations show that this is the case. From the values here employed we can readily compute many other important thermodynamic relations.

TABLE 13.—*Transformation factor from perfect gases to the material of the sun within the photosphere.*

Formula $P_1 = \frac{P_s \rho_h}{\rho_s}$		
	Numbers.	Logarithms.
$P_s$ = surface pressure by Nipher's formula	$2.9004 \times 10^{14}$	14.462460
$\rho_h$ = density of hydrogen at surface of sun	0.000089996	5.954223—10
$\rho_s$ = surface density by Nipher's formula	0.37255	9.571182—10
$P_2$ = corresponding pressure from inside	$7.0065 \times 10^{10}$	10.845501
$P_1$ = pressure found from outside conditions	$7.95967 \times 10^8$	8.900895
$F$ = transformation factor	88.025	1.944606
$R_2$ = gas constant for $P$ from Nipher's formula	$1.0175 \times 10^{11}$	11.007523
$R_1$ = gas constant for $P$ from hydrogen	$1.1559 \times 10^9$	9.062917
$F$ = transformation factor	88.025	1.944606

Some such factor as 88 is required to change the conditions outside the photosphere for perfect gases to those inside the photosphere for nonperfect gases or liquids.

TABLE 14.—*Specific heats  $c_p$ ,  $c_v$ , quantity of heat  $Q$ , and work  $W$ , in the surface stratum of the sun.*

	Numbers.	Logarithms.
$\epsilon_p$ $\left\{ \begin{array}{l} = \frac{\kappa}{\kappa-1} = \frac{3n-4}{2-2n} \\ = \text{assumed value} \end{array} \right.$	3.5 3.4615	0.539264
$A$ = heat equivalent of work	$\frac{1}{4.1855 \times 10^7}$	2.378253—10
$R$ = gas constant	$1.0175 \times 10^{11}$	11.007523
$AR$ =	2431.0	3.385776
$c_p$ = $AR \frac{3n-4}{2-2n} = 3.5 AR$	8414.8	3.925040
Assume 3.4615 $AR$		
$2c_v$	16829.6	
$3AR$	7293.0	
$4AR$	9724.0	
$-\left(\frac{dQ}{dT}\right)_n = + (c_p + 4AR)$ specific heat due to contraction	18138.8	4.258609
$n$ = $\frac{2c_p + 4AR}{2c_p + 3AR} = 1.1$ closely	1.1008	0.041699
$c$ = $\frac{Q}{W} = \frac{c_p + 4AR}{2c_p + 3AR}$	0.7519	9.876184
= 0.75 closely		
$W$ = $0.636 \frac{M^2 k^2}{2r}$ = work of compression	$1.2225 \times 10^{48}$	48.087250
$Q$ = $0.7519 W$ = heat radiated	$0.9192 \times 10^{48}$	47.963434
$W - Q$ = excess of work energy over heat energy	$0.3033 \times 10^{48}$	47.481872
$\frac{Q}{W - Q}$ =	3.03	0.481562
$\frac{W}{W - Q}$ =	4.03	0.605378
$c_v$ = $c_p - \frac{(8-5n)(4-3n)}{3(2-n)^2(5\kappa-3)} AR$	7977.2	3.901850
= $c_p - 0.180 AR$		
$\kappa$ = $\frac{c_p}{c_v} = \frac{8414.8}{7977.2}$	1.0548	0.023190

It may be observed that the Smithsonian Astrophysical Observatory computes from the Washington observations a tem-

perature of about  $6000^{\circ}$  for the atmosphere of the sun, although it is quite certain that a higher station, as Mount Whitney, would give a greater temperature, say  $6500^{\circ}$ . This, of course, takes account of the absorption in the earth's atmosphere, but not of that in the sun's atmosphere. It seems probable that the equivalent of  $1000^{\circ}$  C. may be absorbed from the stratum included between the midst of the photosphere and the top of the inner corona. If this is not the case, then the outgoing radiation of the sun must be such as to give nearly 4.0 gram-calories per square centimeter per minute on the outer surface of the atmosphere of the earth. The relative absorption in the atmospheres of the sun and the earth, respectively, will be much more readily determined if it can be admitted that the temperature of the sun about  $7''$  within the photosphere is approximately  $7652^{\circ}$ . In the following discussion the surface stratum is that which is  $7''$  below the visible boundary of the photosphere, where the pressure is taken as 28.028 atmospheres. The various comments made by Buckingham and Day as to the value of temperatures extrapolated from terrestrial to solar conditions have their importance, but it is believed that we shall be able to gain a footing by other processes, such as thermodynamic relations, and thereby determine the thermal condition of the sun without such an overstepping of the limits of the actual practicable experiments of the laboratory. We will proceed, in Tables 12 to 14, to consider the conditions within the solar mass, with the aid of Nipher's formulæ, and to show that here, too, there is ground for encouragement, because of the numerous agreements between two independent sets of data, namely, the astronomical quantities and the thermodynamic values.

#### DISCUSSION OF THE VALUES DERIVED FROM TABLES 12 TO 14.

Table 12, "Computation of the pressures, temperatures, and densities at the surface and within the sun by Nipher's formulæ," contains a series of values at the surface stratum in the photosphere, where the pressure has been taken at 28.028 atmospheres as the result of external conditions. These have now been computed from astronomical data  $M$ ,  $k^3$ ,  $r$ , and the assumed temperature  $7651.6$ . The purpose is to compare these two sets of values, one computed from external conditions, and the other from the internal conditions, the former for strictly perfect gases, as hydrogen, and the latter from such non-perfect gases or liquid material as makes up the body of the sun. While the law  $Pv = RT$  applies to perfect gases, we may yet obtain some approximate idea of the state of the sun inside the photosphere if a transformation factor can be found by which to pass from the first system to the second. In a circulating mass like the sun it is probable that something like this law applies throughout the mass. At any rate the view can be tested to some extent by studying the two sets of data. There is, of course, some danger of arguing in a circle through so complex a system of formulæ, but I think that the general conditions herein exhibited conform more closely to a natural solar mass than the results heretofore derived by the use of Ritter's formulæ.

#### *The density.*

The average density of the sun from astronomical data is 1.43287, and it is a denser liquid than water. The surface density is 0.37255, or about one-fourth the average density. This latter occurs at the distance  $0.545r$  from the center of the sun, and if anything like the same gradient of density is maintained throughout, the density near the center of the sun is not far from 5.7, which is about the mean density of the earth. We may, therefore, assign a more or less solid nucleus to the sun, which becomes viscous at a distance of about one-third the radius from the center, and soon thereafter mobile. The transitions within the sun are gradual, but at the photosphere there is apparently a mixture of liquid and gaseous masses in active transitions, and these seem to be the conditions indi-

cated by the phenomena observed in the sun spots. The prominences, faculæ, and the chromosphere are strictly in a gaseous atmosphere; the photosphere is a mixture of gases and liquids, and the interior consists of a circulating liquid passing into a solid nucleus near the center. While the sun's pressure by gravitation alone would increase the density of its constituents, the temperature is at the same time high enough to balance this tendency to compression, so that the material in the sun is in about the same state as the material of the earth, except that here the outer layers have advanced toward solidification under the prevailing low temperature. A contracting sun, in order to keep up its radiation, must be circulating freely, and this precludes a very high degree of viscosity, except near the center.

#### *The pressure.*

Beginning with a pressure of 28.028 atmospheres in that layer of the photosphere where the temperature is  $7652^{\circ}$ , which on the sun is equivalent to  $7.96 \times 10^8$  dynes, we compute that for a hydrogen gaseous envelope the pressure practically vanishes at the top of the inner corona. Beyond this layer, into which hydrogen is ejected in the prominences, the conditions are favorable for all the electrical and magnetic phenomena belonging to the cathode rays in rarefied gases. At the photosphere, where the materials change from gases to vapors and liquids, there is a corresponding equivalent increase in pressure up to  $2.90 \times 10^{14}$  dynes. It would take this increase in pressure to pass from the gaseous to the fluid state at the high temperature there prevailing. If a fluid may be considered as a gas brought by pressure at a given temperature to the liquid condition, then this pressure difference also represents the explosive energy when the liquid changes to a gas. If the liquid is elevated from the interior to the surface of the sun by convection currents, then, on reaching the surface, it may greatly expand and even explode when vaporization takes place, as is commonly observed on the edge of the sun through the enormous velocities measured by the change in wave lengths, by the Doeppler principle, or by anomalous dispersion. Within the body of the sun, at the distance 0.5 radius from the center, the pressure is  $1.57 \times 10^{15}$  dynes, which is 5.4 times as much as at the surface. By the same ratio, the pressure would be eleven times as much at the center, though this law doubtless changes within the nucleus. The pressure is comparatively uniform below the sun's surface, and widely discontinuous at the surface. Hence, the convectional currents and the dependent phenomenon of rotation in latitude are leisurely motions compared with the explosive action at the surface layers.

#### *The temperature and the gas constant.*

Nipher's coefficients are carried to only three decimals, which is doubtless sufficiently accurate for the determination of the value of the contractional constant  $n$ . It is not quite sufficiently accurate, however, to give proper check values from one formula to another, but I have not thought it worth while to carry this computation beyond the approximate stage. If we pass from a perfect gas to a fluid, the value of the gas constant adopted must be interpreted as merely suggesting important relations, and too much emphasis must not be laid upon certain obvious criticisms which naturally arise. We may suppose that the mass of the sun beneath the photosphere, while apparently fluid or viscous, yet moves in accordance with the general law, by reason of convection, so that it is continually readjusting itself to conform somewhat closely to this general law of gaseous elasticity. At any rate, this is the theory upon which we have proceeded in the discussion. We compute the product

$R T$  by Nipher's formula, and check it with the product  $\frac{P}{\rho}$  found from the pressure and density, and then with the temperature  $T = 7651.6^{\circ}$  find  $R = 1.0175 \times 10^{11}$  for the fluid of density 0.37255 in the surface layer. The temperature within the sun

at the distance 0.707 radius from the surface becomes  $8264^\circ$ , and at this rate, an increase of  $612^\circ$  in 0.293 radius, the total increase from the surface to the center is  $2089^\circ$ , making the central temperature  $9741^\circ$ . This gives an average gradient of  $-0.0030072^\circ$  per 1000 meters from the center to the surface. We find, also, the gradient from the photosphere to the top of the inner corona to be  $-0.012563^\circ$  per 1000 meters. The gradient of the temperature is about four times as great in the atmosphere of the sun as inside the photosphere. The cooling is, therefore, more rapid outside than it is inside the photosphere.

*The mass of sun, the weight of 1 gram on the surface of the sun, and the transformation factor.*

The mass of the sun is  $2.0091 \times 10^{33}$  by Nipher's formula, agreeing closely with that adopted from Newcomb,  $2.0132 \times 10^{33}$ , the former being computed through the product  $RT$ , and thus checking all the quantities. The weight of 1 gram at the surface of the sun is 27428 by Nipher's formula, through the product  $RT$ , and this agrees with the simple product  $g = 980.6 \times 28.028 = 27484$ , thus checking again. The transition factor from a perfect gaseous system to that actually existing at the surface, where the density is 0.37255, is found as indicated. We find the pressure corresponding to 0.37255 instead of that for which the computation was made in a hydrogen atmosphere of density 0.000089996, and obtain  $P_2 = 7.0065 \times 10^{10}$  through Nipher's formula, as if the atmosphere were of the greater density. For the actual hydrogen atmosphere we computed (Table 13)  $P_1 = 7.95967 \times 10^6$ . Hence,  $P_2 = 88.025 P_1$ , so that 88.025 is the required factor. Similarly, the gas constant from Nipher's formula is  $R_2 = 1.0175 \times 10^{11}$ . It was computed for the actual hydrogen atmosphere (Table 13) to be  $R_1 = 1.1559 \times 10^9$ . Again,  $R_2 = 88.025 R_1$ , so that there is mutual agreement. Some such factor as 88 is required to pass from the law for perfect gases,  $P_1 v = R_1 T_1$ , to that for solar liquids,  $P_2 v = R_2 T_2$ .

It will not be advantageous to speculate as to what this factor 88 signifies, but it is not so large as to be improbable in passing from a gaseous to a fluid state, as it may stand for the internal forces of viscosity or friction and molecular cohesion, and possibly for some unknown forces of electricity and magnetism.

#### *Specific heats, energy of radiation, and contraction.*

Carrying the values of the several quantities through the various formulæ we find that they conform to the prescribed conditions, as follows:

Specific heat of contraction	$-\left(\frac{dQ}{dT}\right)_n$	=	18138.8
Exponent and coefficient	$n$	=	1.1 closely.
Heat energy of radiation	$Q$	=	$0.9192 \times 10^{10}$
Work energy of contraction	$W$	=	$1.2225 \times 10^{10}$
Ratio $\frac{\text{heat radiated}}{\text{work of gravitation}}$	$= c = \frac{Q}{W}$	=	0.75 closely.
Ratio $\frac{\text{heat radiated}}{\text{excess}}$	$= \frac{Q}{W - Q}$	=	3.00 closely.
Ratio $\frac{\text{work of compression}}{\text{excess}}$	$= \frac{W}{W - Q}$	=	4.00 closely.
Specific heat at constant pressure	$c_p$	=	8414.8
Specific heat at constant volume	$c_v$	=	7977.2
$\kappa = \frac{c_p}{c_v}$ ratio of the specific heats at the temperature $7652^\circ$		=	1.0548

We note that this ratio  $\kappa = \frac{c_p}{c_v} = 1.4065$  in terrestrial conditions; in solar conditions inside the photosphere  $\kappa = 1.0548$ ; and in the hydrogen envelope  $\kappa = 1.000052$  according to the preceding discussion.

Surveying this set of interrelated thermodynamic values, and especially in view of the fact that they seem to conform so well with the known astrophysical conditions derived from

observation, and with the astronomical data obtained by the general laws of motion, we conclude that they afford ground for further research. If they form the approximate basis for a sound solar physics they will become important in further meteorological studies.

#### THE TEMPERATURE ELEMENT OF THE CLIMATE OF BINGHAMTON, N. Y.

By W. E. DONALDSON, Observer, Weather Bureau.

[Condensed from a paper read before the Binghamton Academy of Science on March 1, 1904.]

The climate of Binghamton is continental; the climate of Iceland is oceanic; the climate of Omaha, Nebr., is continental. As a result the January climate of the coast of Iceland is  $11^\circ$  warmer than the January climate of Omaha, and  $7^\circ$  warmer than the January climate of Binghamton. The July climate of the coast of Iceland is  $26^\circ$  cooler than the July climate of Omaha, and  $21^\circ$  cooler than the July climate of Binghamton. Thus the Binghamton climate occupies an intermediate position between the climate of Omaha and that of Iceland; though differing very slightly from the climate of Omaha, it differs decidedly from that of Iceland.

The normal mean temperature, by decades, has its minimum,  $21^\circ$ , in the first decade of February and its maximum,  $72^\circ$ , in the first decade of July.

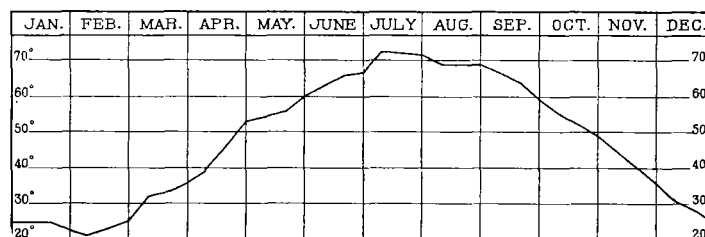


FIG. 1.—Normal annual temperature curve at Binghamton, N. Y., determined from seven years' record, October 1, 1896–September 30, 1903.

The change from winter to summer in this section is decidedly more rapid than the change from summer to winter. The normal annual temperature curve, fig. 1, is ascending 150 days and descending 215 days. This curve has a marked resemblance to the normal diurnal temperature curve in summer, fig. 2. The rapid rise from February 10 to July 10 resembles very closely the rapid rise from sunrise until about 2 p. m.; the slight change from July 10 to September 10 resembles the slight change from about 2 p. m. to 5 p. m.; the rapid fall from September 10 to December 31 resembles the rapid fall from about 5 p. m. to about 3 a. m., and the slight change from December 31 to February 10 resembles the slight change from about 3 a. m. to sunrise.

In the summer the diurnal temperature changes are in accordance with the diurnal variation in the intensity of insolation. The minimum temperature usually occurs at sunrise and the maximum about 3 p. m. The mean temperatures for individual summers closely approximate the normal for the summer. The regularity of the diurnal temperature curve day after day in summer and the close approximation of the mean temperatures of each summer to the normal summer temperature, result from the nonimportation of large masses of air from distant points.

In the winter the diurnal temperature curve, figs. 3, 4, and 5, frequently has no similarity to the diurnal variation in inso-

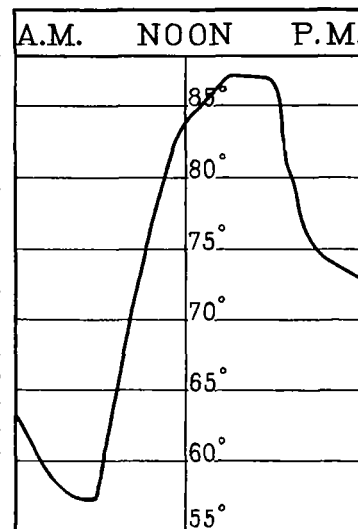


FIG. 2.—Summer type. Diurnal temperature curve on June 25, 1901.